

A Hierarchical Counts-in-Cells Model

Incorporating Halo Occupation Scatter For Radio Surveys

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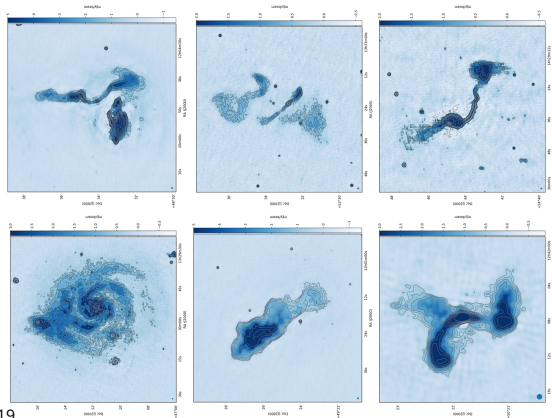
Bielefeld University
LOFAR Cosmology Group

LOFAR Annual Meeting
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Radio galaxies

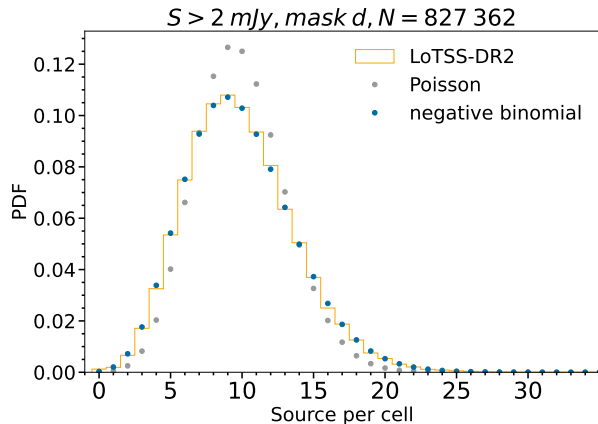
- Radio surveys offer an independent window
- Not suffering from dust extinction, reddening, stellar contamination
- Probing structure growth at intermediate redshifts
- Differently from early-universe CMB probes and low- z optical lensing
- Wide area coverage
- Deep redshift reach up to $z \approx 7$ enough to allow for several tests that so far could only be done with IR/optical surveys



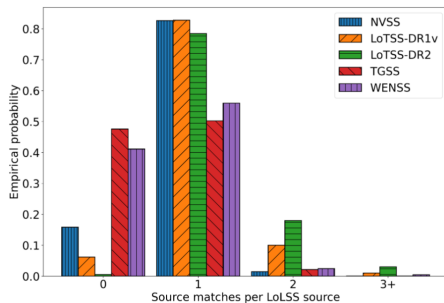
Shimwell et al, 2019

Distribution of the radio sources

(results of LoTSS-DR2)



Pashapour-Ahmadabadi et al, 2025



Böhme et al, 2023

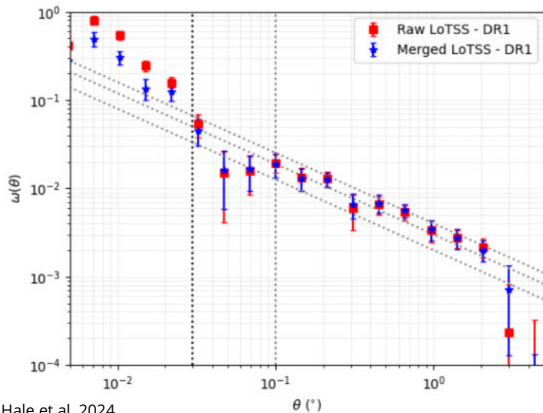
$$\text{NegativeBinomial}(y; r, p) \propto (1 - p)^r p^y$$

Mean number of components per source ≈ 1.27

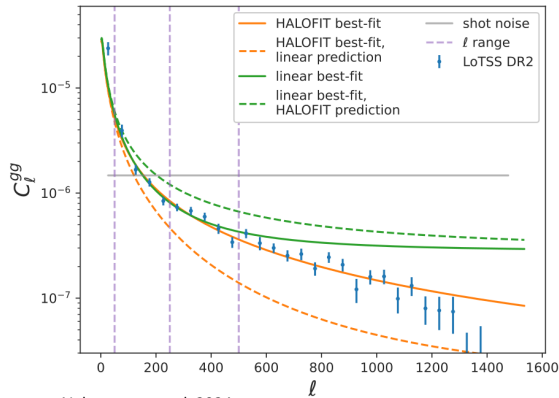
in agreement with Böhme et al (2023)

Clustering of the radio sources

(results of LoTSS-DR2)



Hale et al, 2024



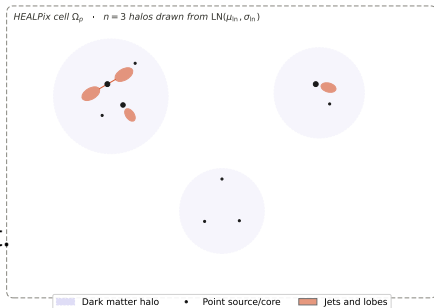
Nakoneczny et al, 2024

Halo Model

- Halofit model:
 - 1-halo effect: clustering within the same halo +
 - 2-halo effect: clustering of galaxies in different halos
- Model matter density field $\rho(\mathbf{x})$ as a log-normal dist. (Coles & Jones 1991)

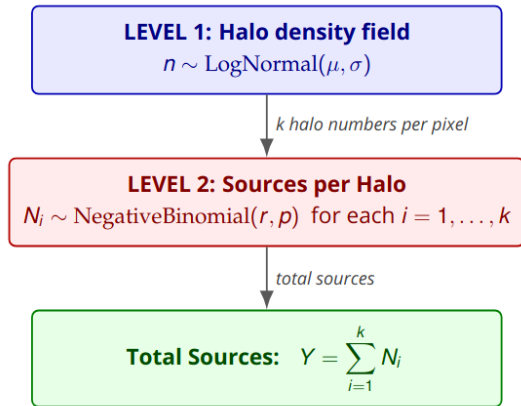
$$\text{LogNormal}(x; \mu_{\ln}, \sigma_{\ln}) \propto \exp \left[-\frac{(\ln x - \mu_{\ln})^2}{2\sigma_{\ln}^2} \right]$$

- At early times — when density fluctuations are small — the log-normal naturally reduces to Gaussian linear theory.
- The $\sigma_{\ln}$ directly encodes the variance of the halo field $\sigma_{\ln}^2 \approx b^2 \sigma_m^2$



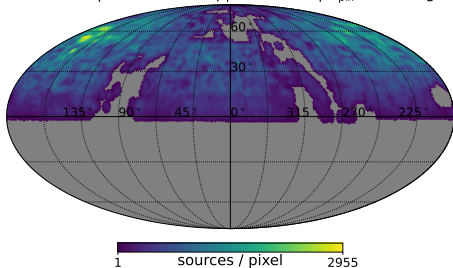
Hierarchical Model: LogNormal-NegativeBinomial

- The number of halos per pixel n traces the underlying density field
- In each halo the radio galaxies follow negative binomial distribution

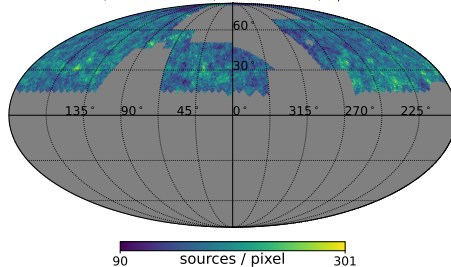


LoTSS DR3

LoTSS DR3 | raw counts map | no flux cut | $\Omega_{pix} = 0.84 \text{ deg}^2$



LoTSS DR3 | Spatial mask | flux > 2 mJy | $\Omega_{pix} = 0.84 \text{ deg}^2$



- Angular resolution of 6" (9" below declination 10°)
- 13,667,877 sources
- Median sensitivity of 92 $\mu\text{Jy}/\text{beam}$
- Spatial mask (generated by Lukas Böhme) excludes the 9" regions & high noise regions

Model Test

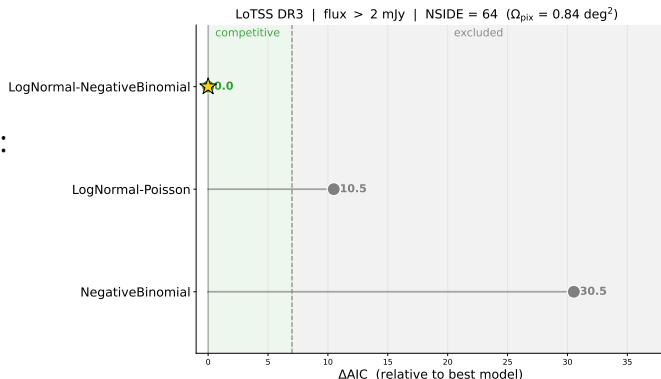
- Akaike information criterion (AIC):

$$AIC = 2k - 2 \ln(\mathcal{L})$$

$$\Delta AIC = AIC_i - AIC_{\text{best}}$$

(Exclusion limits from Burnham & Anderson 2002)

- LogNormal-NegativeBinomial distribution is strongly favoured!
- Mean number of components per source ≈ 2
consistent with negative binomial
- Mean number of halos per pixel ≈ 4



Connection to matter variance and halo density

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$$\sigma_{\ln}^2(\Omega_{pix}) = b^2 \sigma_m^2(\Omega_{pix})$$

$$\sigma_m^2(\Omega_{pix}) = \sum_l \frac{2l+1}{4\pi} C_l W_l^2(\Omega_{pix}), \quad C_l = \int dz \frac{H(z)}{c\chi^2(z)} P(k, z) p^2(z)$$

$p(z)$ from three LOFAR deep fields and their multiwavelength data (Hale et al, 2024)

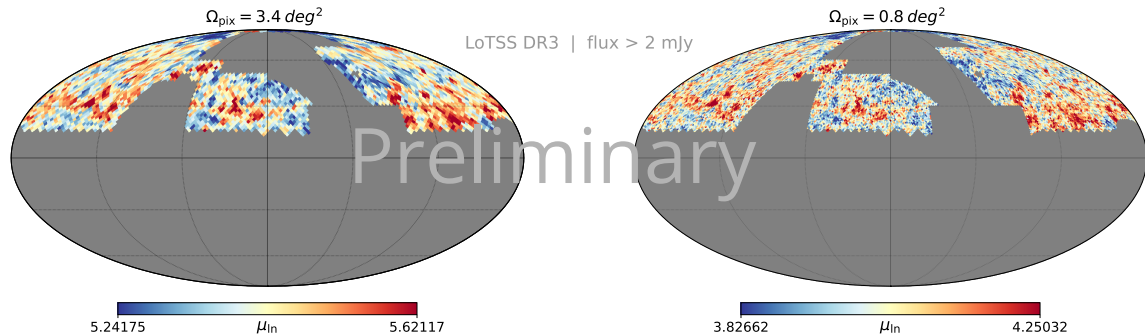
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$$\mu_{\ln}(\Omega_{pix}) = \log(n_{\text{halo}} \Omega_{\text{pix}} \frac{dV}{d\Omega}) - \frac{1}{2} \sigma_{\ln}^2, \quad \frac{dV}{d\Omega} = \int dz \frac{c}{H(z)} \chi^2(z) p(z)$$

All cosmological parameters from Planck 2018

- Four shared physical parameters $\{n_{\text{halo}}, b, r, p\}$
- Jointly fitting for different pixel sizes

Mean Halo Field, LoTSS-DR3



- Reconstructed mean halo density field can be cross-correlated with:
 - CMB lensing convergence
 - weak-lensing shear probes

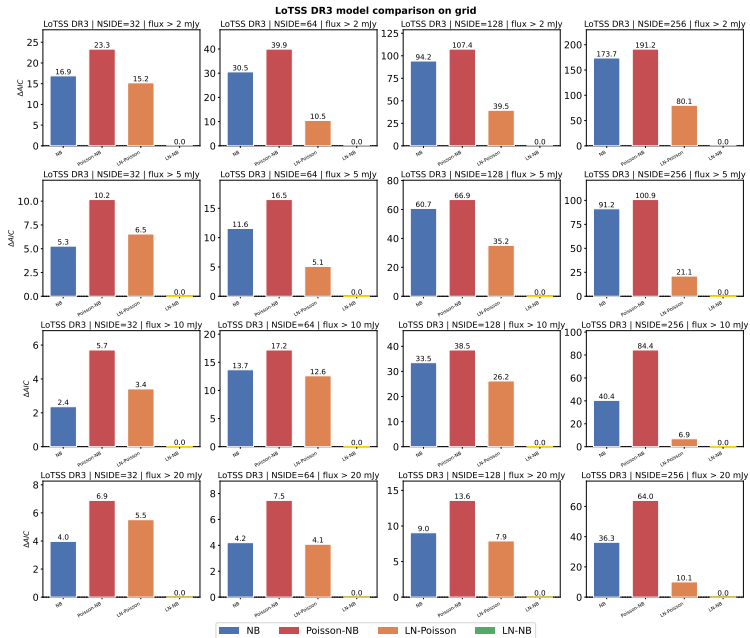
Summary

- The LN-NB compound distribution provides a new hierarchical model, incorporating halo occupation scatter for radio surveys.
- It fits excellent to LoTSS-DR3 source counts.
- It can be used to map the underlying dark matter field.
- The per-pixel lognormal mean $\mu_{\ln}(i) = \log \mathbb{E}[K|y_i] - \frac{1}{2}\sigma_{\ln}^2$ constitutes a Bayesian reconstruction of the projected halo density field.
- The reconstructed halo density field can be then cross-correlated with CMB lensing and weak lensing probes.

Backup slides

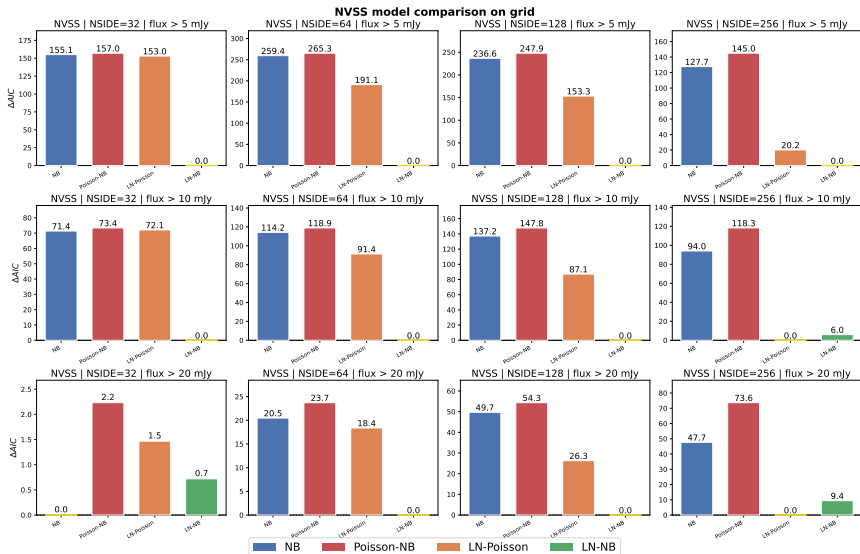
Results

LoTSS-DR3

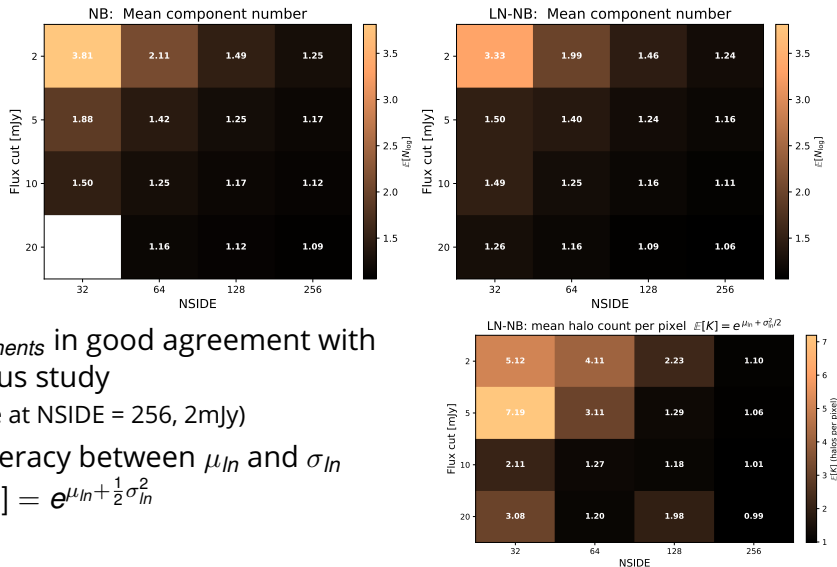


Results

NVSS



Mean component and halo numbers



- $\mu_{components}$ in good agreement with previous study (e.g. see at NSIDE = 256, 2mJy)

- Degeneracy between μ_{ln} and σ_{ln}

$$\mathbb{E}[N_{halo}] = e^{\mu_{ln} + \frac{1}{2}\sigma_{ln}^2}$$

Distribution characteristics

- $\mu_{\ln}, \sigma_{\ln}$: log-mean and log-scatter of the halo field distribution.
- $\mathbb{E}[X] = e^{\mu_{\ln} + \sigma_{\ln}^2/2}$
- r : NB dispersion parameter per halo, encoding multi-component variance arising from radio galaxies morphology
- p : NB success probability, encoding the mean number of radio sources per halo as $\mu_{NB} = rp/(1 - p)$, and mean number of components $\mu_L = -p/[(1 - p) \ln(1 - p)]$

Halo density

